4. Modelling

Objectives of simulations include the prediction of system performance, such as stability analysis, the identification of potential problems and, ultimately, the establishment of appropriate solutions to the identified problems. Two basic approaches exist: time-domain modelling and frequency-domain modelling.

Time-domain simulation is the most widespread method for system analysis. It is well established, and various simulation tools with user-friendly interfaces exist. They are generally circuit-oriented, i.e. the user only supplies the interconnection of the circuit element models. Time-domain simulation methods are suitable for both large and small-signal analysis for linear and non-linear systems. With the simulation method in the time-domain, disturbances are applied, system responses are calculated and dynamics are observed through plotted curves. Thus, this simulation method presents only results but no explanation to the phenomenon. Furthermore, time-domain simulations tend to be very slow for two reasons: Firstly, the analysis of stiff systems requires small simulation steps over a long period of time. Secondly, many different disturbances have to be applied in order to obtain as many responses as possible.

On the other hand, frequency-domain modelling is a systematic approach which reveals rules behind complicated phenomena of system dynamics. As a consequence, there is no need to apply many different disturbances. However, the frequency-domain modelling approach presented in this thesis requires linear or linearised models and is in most cases confined to small-signal analysis. Furthermore, it fails to represent transient processes.

Finally, it should be mentioned that time and frequency-domain modelling approaches are suitable for mixed-signal analysis. Furthermore, both methods have in common that computers can only simulate systems at discrete points in time or frequency spaced Δt or Δf apart, resulting inevitably in truncation errors.

4.1. Time-domain modelling

Time-domain modelling is well established and many programs exist, such as PSCAD/EMTDC, NETOMAC, SIMPLORER, PSpice, etc. Because of their widespread use and the countless number of publications on this subject, this modelling approach is covered only very briefly. Special emphasis is given to the program PSCAD/EMTDC, as this program is used to verify all results obtained through the proposed modelling approach in the frequency-domain.

PSCAD/EMTDC[1] is a fixed time step program where the time step is chosen before the start of the simulation. Transient simulation of an electrical network is accomplished by solving the network equations at a series of discrete intervals (time steps). Due to the fixed nature of the time step, switching events that occur at any time after a time step interval cannot be accounted for and represented until the following time step. This phenomenon can introduce inaccuracies and is aggravated in case of fast switching which is always the case for PWM controlled voltage source converters. There are several ways to overcome this problem:

- Reducing the time step which inevitably increases the computation time and the size of output data files accordingly and may still not be accurate enough.
- Using a variable time step solution where the program will subdivide the time step into smaller intervals if a switching event is detected. Data files are hard to post-process because of non-uniform sampling.
- Finally, it is possible to use interpolation algorithms to find the exact instant of the event if the switching event occurred between time steps. Once the switching event is accurately calculated, the program continues the normal solution routine. PSCAD/EMTDC offers the possibility to use such an interpolation algorithm, which is one reason for using this program [76].

Any linear time-periodic (LTP) system can be represented by the general state-space equation

$$\begin{aligned} \frac{\mathrm{d}\,[x(t)]}{\mathrm{d}t} &= [A(t)]\,[x(t)] + [B(t)]\,[u(t)] \quad \Leftrightarrow \quad \dot{\mathbf{x}} = [A(t)]\,\mathbf{x} + [B(t)]\,\mathbf{u} \\ [z(t)] &= [C(t)]\,[x(t)] + [D(t)]\,[u(t)] \quad \Leftrightarrow \quad \mathbf{z} = [C(t)]\,\mathbf{x} + [D(t)]\,\mathbf{u} \end{aligned} \tag{4.1}$$

$[x(t)] = \mathbf{x}$ is called *state vector*, $[u(t)] = \mathbf{u}$ is called *input vector* and $[z(t)] = \mathbf{z}$ is called *output vector*. The first equation is the *state-space equation* and the second equation is the *output equation* where the *coefficient matrices* $[A(t)]$ to $[D(t)]$ are all T-periodic. The matrix $[A(t)]$ is called *state matrix* , the matrix $[B(t)]$ is called *control matrix*, the matrix $[C(t)]$ is called *output matrix* and the matrix $[D(t)]$ is called *feed-forward matrix*. The block diagram of the state-space representation is shown in Figure 4.1 [77].

The full state-space equation is not required for some very common power electronic elements. It is simplified to $[z(t)] = [D]\,[u(t)]$ for ohmic resistances, to $\frac{\mathrm{d}[x(t)]}{\mathrm{d}t} = [B]\,[u(t)]$ for capacitors and inductors and to $[z(t)] = [D(t)]\,[u(t)]$ for voltage source converters, as it is shown in Chapter 5. As far as the scope of this thesis is concerned, induction machines are the only systems whose state-space representation is not that greatly simplified.

[1] EMTDC is a numerical equation solver that is most suitable for simulating time-domain responses, also known as electromagnetic transients and hence the name electromagnetic transients program (EMTP). PSCAD is the graphical user interface.

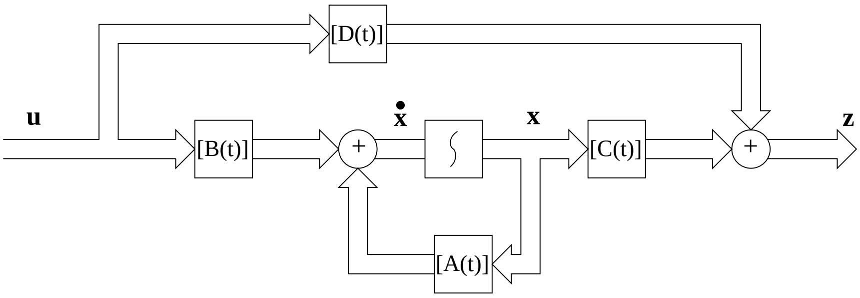

Figure 4.1: Block diagram of the state-space representation

Numerical integration methods The system of linear differential equations can be written more compactly as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) . \tag{4.2}$$

Various methods have been developed over the years to numerically solve this set of equations. Common methods include Backward Euler, the Trapezoidal method, Runga-Kutta methods etc. They all differ in terms of accuracy, stability and the speed of execution. Details on these methods can be found in [21, 78].

Non-linear differential equations Typically, differential equations for power electronic circuits are extremely non-linear and stiff. The non-linear set of differential equations can be written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), t) . \tag{4.3}$$

Solving this set of equations requires iterative procedures, such as the Newton-Raphson procedure, resulting in very long simulation runs [21]. A significant amount of research has been spent on fast algorithms for finding the steady-state solution for time-periodic equations. The steady-state solution algorithm as proposed by Aprille and Trick is based on the two-point boundary value problem and uses Newton's method [79].

4.2. Frequency-domain modelling using frequency coupling matrices

Frequency-domain modelling is often understood as eigenvalue or modal analysis. The modelling approach in the frequency-domain presented in this thesis uses the concept of operational matrices. It refers to the manipulation of coefficients that describe a function $f(t)$ rather than working with $f(t)$ itself. The coefficients for each function are condensed into vectors or matrices. Operations on $f(t)$, such as modulation in converter circuits, attenuation, rectification etc., are accomplished by studying how the coefficients of $f(t)$ change as the function undergoes the given operation.